WJEC CBAC

GCSE LINKED PAIR PILOT

4361/02

APPLICATIONS OF MATHEMATICS
UNIT 1: APPLICATIONS 1
HIGHER TIER

A.M. MONDAY, 16 January 2012

2 hours

ADDITIONAL MATERIALS

A calculator will be required for this paper.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded that assessment will take into account the quality of written communication (including mathematical communication) used in your answer to question **3**(*b*).
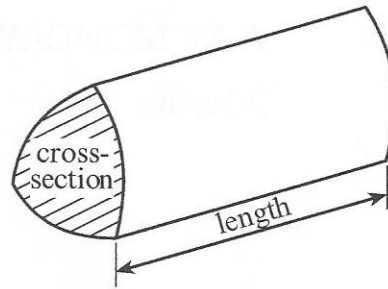
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	8	
2	7	
3	12	
4	14	
5	10	
6	10	
7	5	
8	7	
9	7	
10	5	
11	3	
12	12	
TOTAL MARK		

Formula List

Area of trapezium $= \frac{1}{2}(a+b)h$
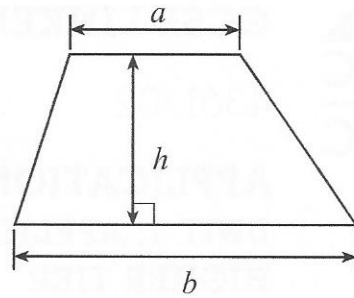
Volume of prism = area of cross-section × length

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
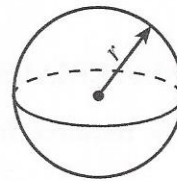
Volume of cone $= \frac{1}{3}\pi r^2 h$

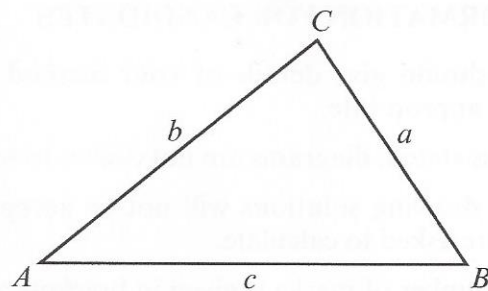**Curved surface area of cone** $= \pi r l$
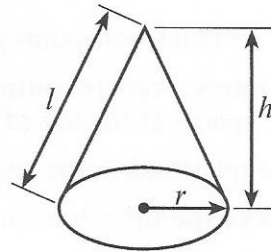
In any triangle ABC

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \dfrac{1}{2}ab\sin C$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

1. Write down expressions for each of the following.

(a) The total cost of 10 pencils at *g* pence each and 5 pens at *h* pence each.

$$\text{Total Cost} = 10g + 5h$$

[2]
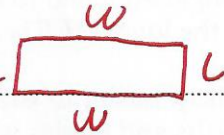
(b) The mean height of the three boys listed below.

	Height in cm
Adam	x
Tommy	y
Joseph	z

$$\text{Mean} = \frac{x + y + z}{3}$$

[2]

(c) The perimeter of a rectangle with length *l* cm and width *w* cm.

$$\text{Perimeter} = w + w + l + l = 2w + 2l$$

[2]

(d) The smaller share when £*q* is shared in the ratio 1:3.

$$\text{Sum of ratio} = 1 + 3 = 4$$

$$\Rightarrow \text{Smaller share} = \frac{q}{4}$$

As smaller share

[2]

Turn over.

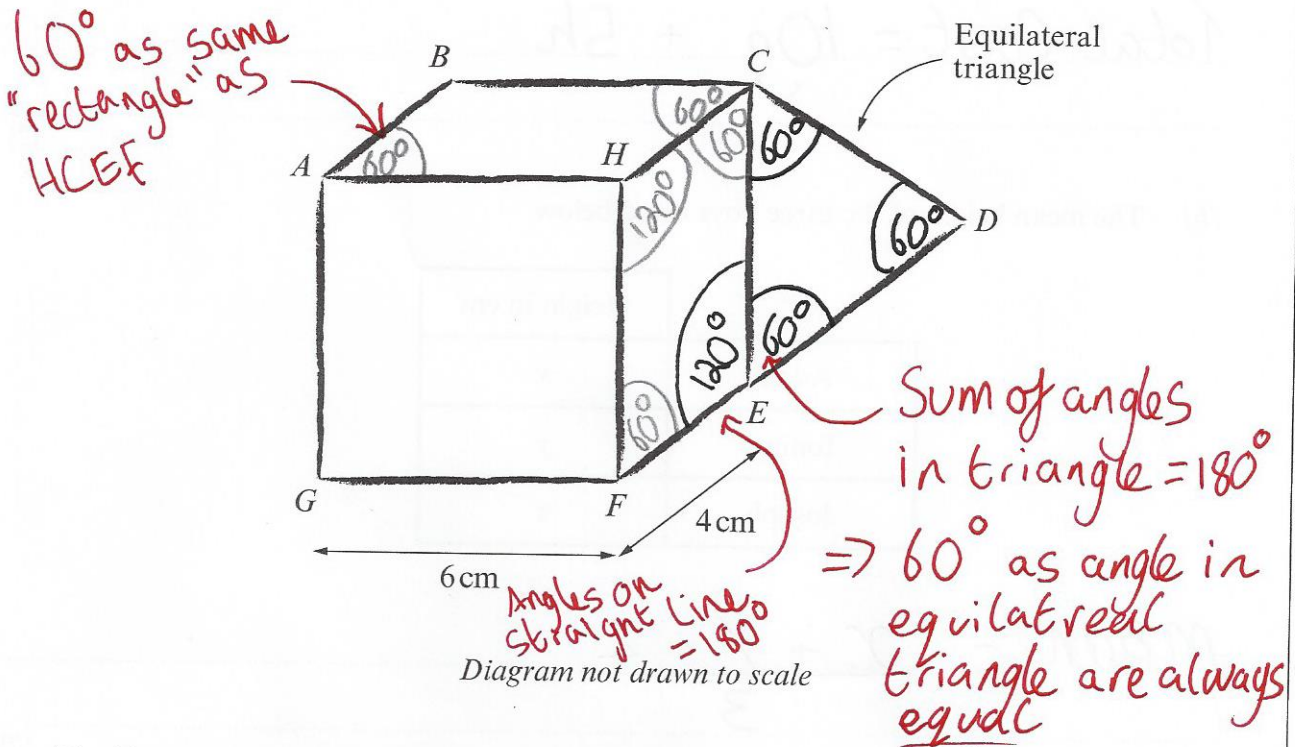**2.** Sasha has designed a new two dimensional logo for her company.
The logo is to be printed on all official company paper.
A sketch of the logo is shown below.

Handwritten annotations:
60° as same "rectangle" as HCEF

Equilateral triangle

Sum of angles in triangle = 180°
=> 60° as angle in equilateral triangle are always equal

Angles on straight Line = 180

6 cm

4 cm

Diagram not drawn to scale

The idea for the logo is based on a 2-D representation of a cube with an accurate equilateral triangle attached to one edge.
On the logo, **DEF is a straight line**.

To help Sasha to draw an accurate version of the logo, she needs to identify what the actual lengths and angles will be on the logo.

(a) Write down the actual

size of $C\hat{E}D$ on the logo ......**60**..... °

length of CE on the logo ......**6**..... cm

length of AB on the logo ......**4**..... cm.

[3]

(b) Complete the accurate drawing of the logo on the opposite page.
AHFG has been drawn for you.

[4]

Handwritten:
HCEF
confusion: opposite angles in quadrilatreal are equal. As drawn on above. Angle H and E are equal

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All sides of triangle are 6cm as equilatreal triangle

← 6cm →

← 4cm →

60°

A

← 6cm →

C

60°

← 6cm →

6cm

H

130°

← 6cm →

6cm

← 4cm →

G

F

60°

← 4cm →

E

← 6cm →
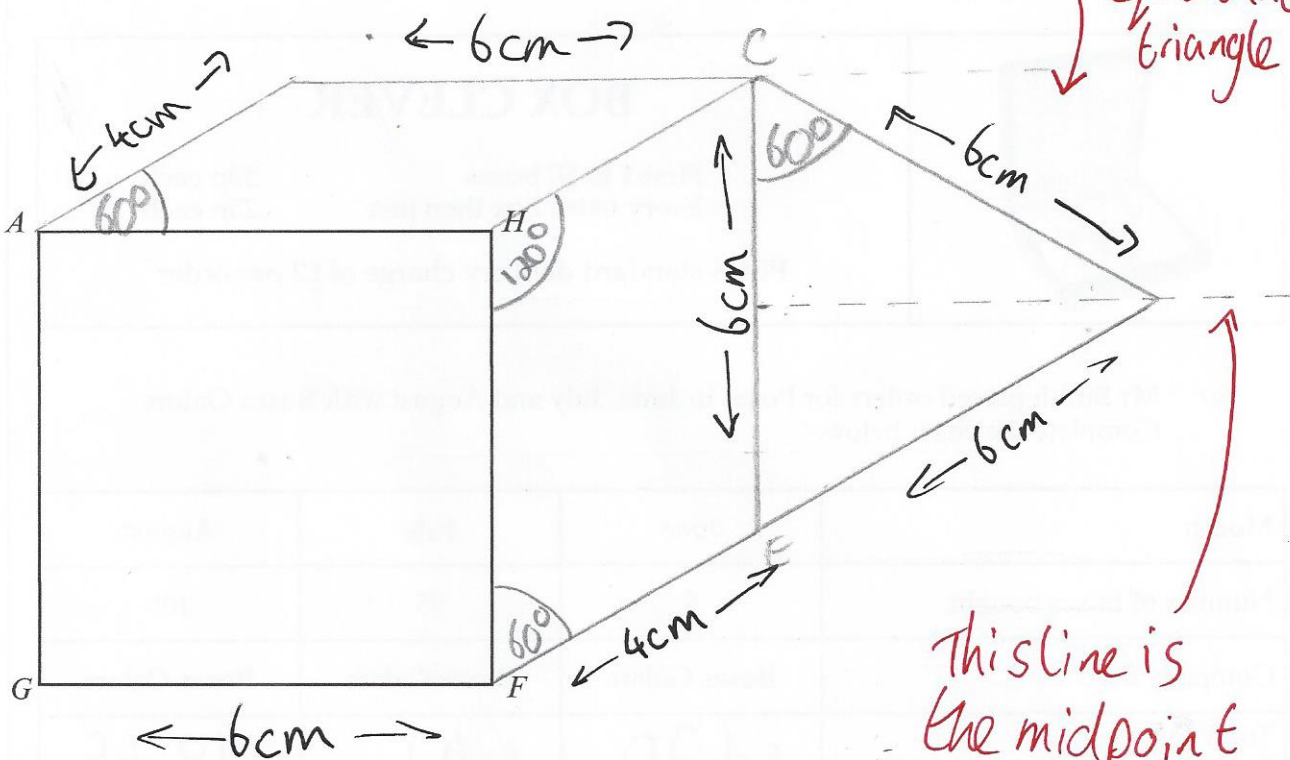
This line is the midpoint of CE. allows us to form equilatreal triangle.

Turn over.

3. Mr Smith needs to buy boxes for necklaces that he makes.
There are two possible suppliers he can use, Boxes Galore or Box Clever.

BOXES GALORE

Order 1 to 100 boxes pay 40p for each box

Need more than 100 boxes? .... Special Offer:

MORE than 100 boxes get **all boxes for just** 15p each!

PLUS standard delivery charge of £3 per order.

BOX CLEVER

First 1 to 50 boxes   30p each
Every extra box then just   25p each

PLUS standard delivery charge of £2 per order

(a) Mr Smith placed orders for boxes in June, July and August with Boxes Galore.
Complete the chart below.

Month	June	July	August
Number of boxes bought	8	95	105
Company used	**Boxes Galore**	**Boxes Galore**	**Boxes Galore**
Total cost (including the delivery charge)	£ 6.20	£ 41	£ 18.75

June = (8 × 0.4) + 3 = £6.20
July = (95 × 0.4) + 3 = £41
August = (105 × 0.15) + 3 = £18.75

[4]

(b) *You will be assessed on the quality of your written communication in this part of the question.*

Mr Smith notices that he has not been wise with his order from Boxes Galore.
In September he intends to buy 96 boxes from Boxes Galore.
What advice would you give Mr Smith?
You must give a reason for your answer.

My advice would be to buy 101 boxes as you save more money and get 5 extra boxes.
Price of 96 boxes = $(96 \times 0.40) + 3 = £41.40$
Price of 101 boxes = $(101 \times 0.15) + 3 = £18.15$
Therefore you make a saving of £23.25 and have 5 extra boxes to store necklases in.
Price per box is reduced when more than 100 boxes are purchased.

[4]

$41.40 - 18.15 = £23.25$

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Turn over.

(c) Use the graph paper below to show the costs for ordering up to 130 boxes from **Box Clever**.

Price of first 50 = 50 × 0.30 = £15   (£17) } 20
Price of next 80 = 80 × 0.25 = £20   (£37) difference

As base delivery charge of £2
=> Starts at 0 boxes = £2

£2 delivery

[3]

Cost in £s, including delivery charge

Buying from **Box Clever**

Number of boxes

(4361-02)
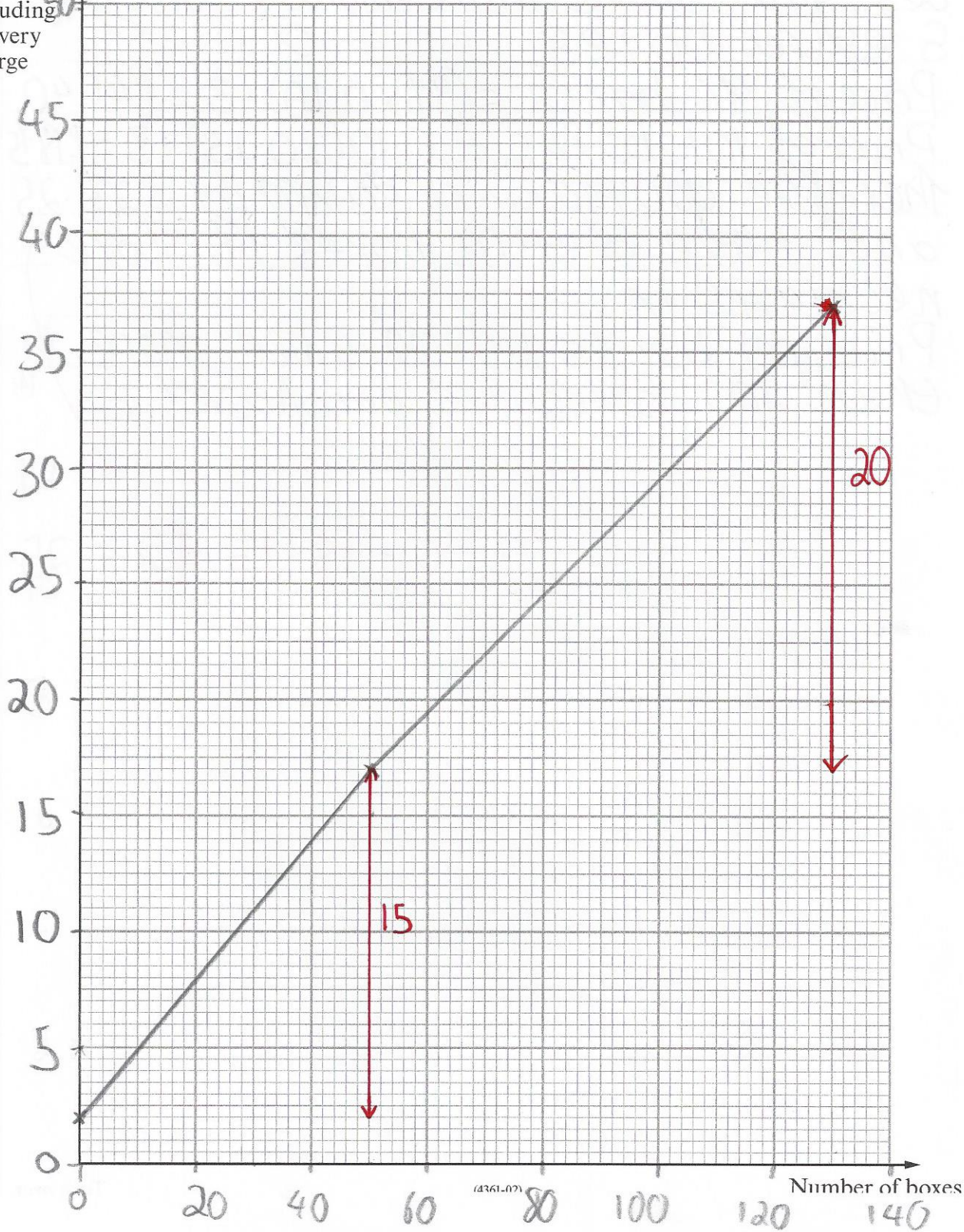
(d)   Complete the chart below to give recommendations for Mr Smith.

Number of boxes	Better company to buy from
Fewer than 8	Box Clever
Between 50 and 54	Box Clever
More than 110	Boxes Galore

Fewer than 8, Box Clever as 30p per box
50 → 54, Box clever as 25p per box
More then 110, Boxes Galore as 15p per box
[1]

Turn over.

4. A machine is used to pack boxes of chocolate beans.
To check the machine, 10 boxes of beans are selected on the hour for 10 consecutive hours.
There should be exactly 55 chocolate beans in each box.
Each hour the number of boxes containing exactly 55 chocolate beans is recorded.

Time	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00
Number of the 10 boxes with exactly 55 beans	8	7	6	9 ✳	8 ⭕	10	8	6	9	9

(a) Is a statement on the box that says

"Contains at least 55 chocolate beans"

always true?
You must give a reason for your answer.

This information was not recorded, doesn't state more or less than 55. Therefore conclusion: NO

[2]

(b) If the experiment were to be carried out again would you expect the results to be exactly the same?
You must give a reason for your answer.

There appears to be no repeating pattern in the readings per hour therefore there will be different readings in a repeating test. conclusion: NO

[2]

(c) It is decided to record and plot the relative frequencies for the information shown in the previous table.

✳ 21 + 9   ⭕ 30 + 8   etc.

(i) Complete the table below.

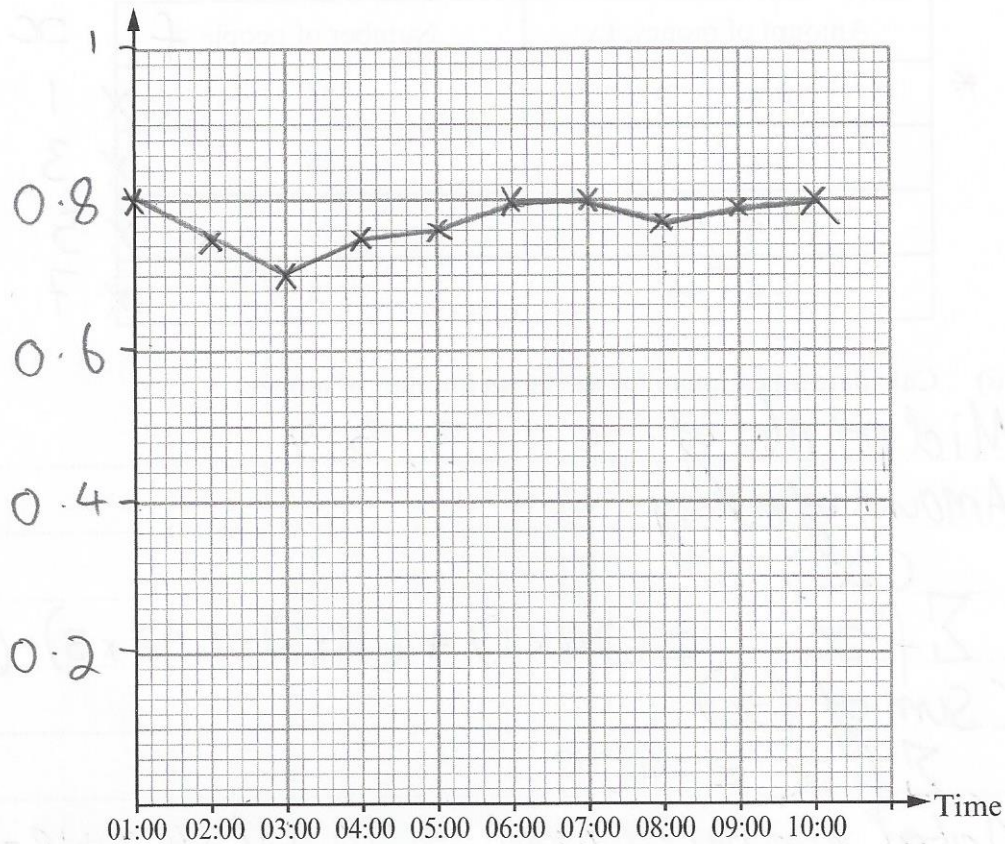| Time, by	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00
Total number of boxes with exactly 55 beans	8	(15)	21	30	38	48	56	62	71	80
Total number of boxes checked	10	(20)	30	40	50	60	70	80	90	100
Relative frequency	0·8	0.75	0.7	0·75	0.76	0·80	0·80	0·775	0·789	0·8

[4]

$\frac{8}{10}$ $\frac{15}{20} = 0.75$ etc.

(4361-02)

(ii) Use the graph paper below to plot the relative frequencies.

Relative frequency

[3]

(iii) Write down the best estimate for the probability that a box selected at random will contain exactly 55 chocolate beans. Give a reason for your answer.

0.8

As the relative frequency gets closer to the probability as the number of trials increases, the last point plotted is 0.8.

[2]

(iv) How would you improve on your estimate?

Repeated experiment many times to then compare result.

[1]

Turn over.

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5. *(a)* A number of people were asked how much they would be willing to pay to go on a fairground ride.
The results are summarised in the table.

Amount of money, £x	Number of people f	x
$0 \leqslant x < 2$	24	1
$2 \leqslant x < 4$	16	3
$4 \leqslant x < 6$	42	5
$6 \leqslant x < 8$	18	7

(handwritten annotations: midpoints 1, 3, 5, 7 marked above the class intervals; ✻ next to first row)

(i) Calculate an estimate for the mean amount of money.

Mid points of Amount of money (x) = 1, 3, 5, 7

$\Rightarrow \sum f x$ (Sum of fx) $= (24 \times 1) + (16 \times 3) + (42 \times 5) + (18 \times 7)$

$\therefore \sum f x = 408$

Total number of people $= 24 + 16 + 42 + 18 = 100$

Mean amount of money $= \dfrac{\sum f x}{\text{Total num. people}} = \dfrac{408}{100}$

$= £4.08$

[4]

(ii) Find the greatest possible value of the range.

AS $x < 8$ and unit is money x can equal £7.99.

$\therefore$ Range $= 7.99$

[1]

(handwritten note:) Range = Biggest − Smallest
Value   Value
$= 7.99 - 0 \Rightarrow 0$

(handwritten note:) ✻ $0 \leqslant x$
Means x can equal 0 (Smallest Value)

(4361-02)

Examiner
only

(b) The fairground owner recorded the number of people coming into his fairground during the different seasons.

Season	• Winter 2010	• Spring 2010	•Summer 2010	• Autumn 2010	Winter 2011	Spring 2011	Summer 2011
Number of people	234	156	316	230	326	456	324

(handwritten above columns: 1, 2, 3, 4)

(i) Calculate the 4-point moving averages and complete the table below.

• Autumn 2010 $= \dfrac{234 + 156 + 316 + 230}{4} = 234$

~~Ackt~~ Winter 2011 $= \dfrac{156 + 316 + 230 + 326}{4} = 257$

Spring 2011 $= \dfrac{316 + 230 + 326 + 456}{4} = 332$

Summer 2011 $= \dfrac{230 + 326 + 456 + 324}{4} = 334$

(handwritten margin note: Average of 4 previous points)

4-point period ending:	Autumn 2010	Winter 2011	Spring 2011	Summer 2011
4-point moving average	234	257	332	334

[3]

(ii) The fairground owner looks at the moving average with his staff. Explain why considering the moving average is worthwhile.

Moving average smooths out the sesonality data, giving a clear overview of the season.

[1]

(iii) The fairground owner states
"we were as busy in Summer 2011 as we were in Spring 2011".
Explain fully the mathematical error in his statement.

This statement is not true when looking at the raw data.

[1]

Turn over.

6. The table gives a grouped frequency distribution of the arm lengths of 100 women each measured correct to the nearest centimetre.

Arm length, a cm	156 to 158	159 to 161	162 to 164	165 to 167	168 to 170
Number of women	5	15	35	40	5

(a) Complete the following cumulative frequency table.

x

y

Arm length, a cm	a < 155.5	a < 158.5	a < 161.5	a < 164.5	a < 167.5	a < 170.5
Cumulative frequency	0	5	20	55	95	100

Always use highest value for x axis E.g (155.5, 0)

15+5   20+35   55+40   95+5   [1]

(b) On the graph paper below, draw a cumulative frequency diagram to show this information.
[2]

Cumulative *x* *y* frequency

161.8   164.2   166

Arm length in cm
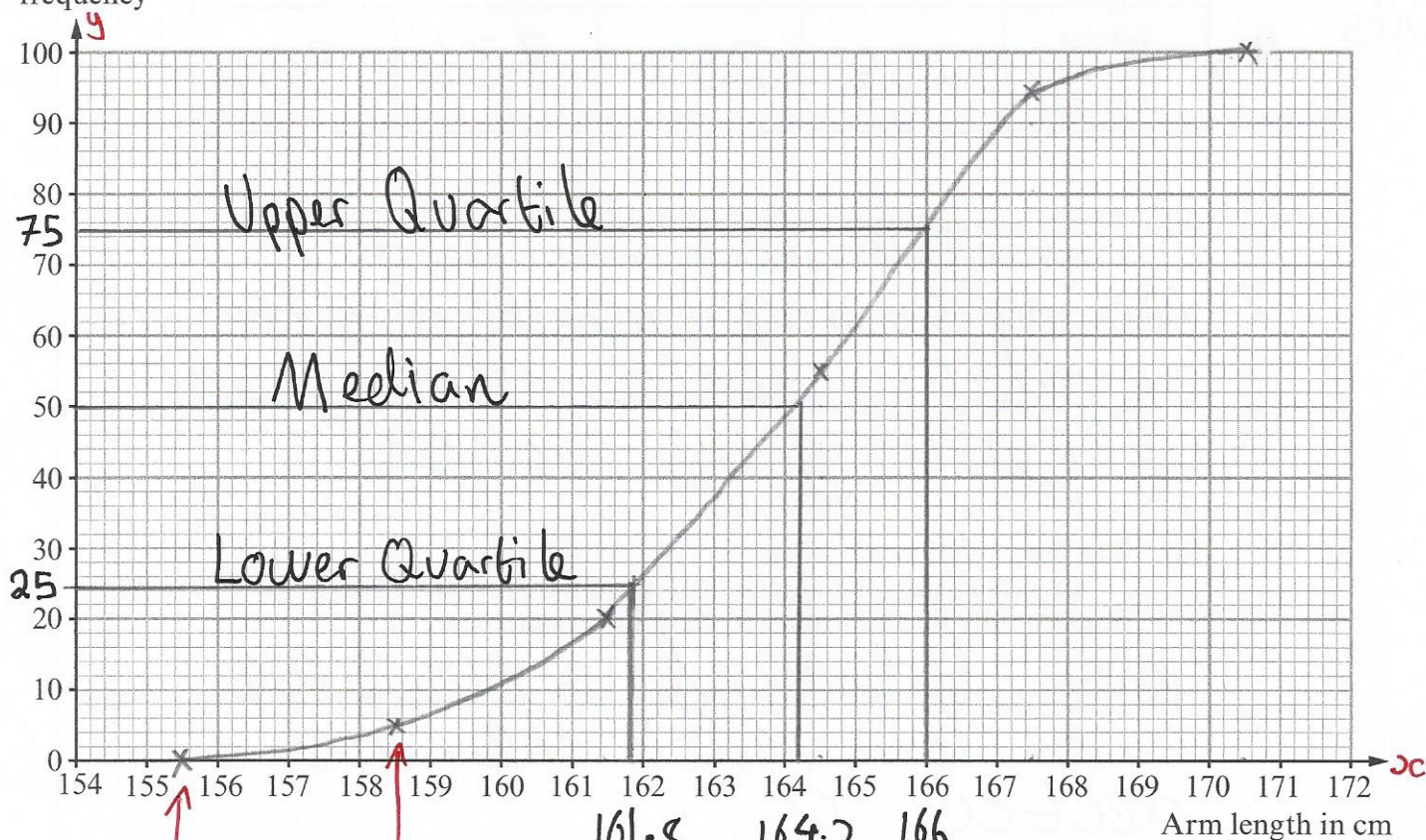
(155.5,0)   (158.5,5) etc.

middle
= 7 ½ way up

$\frac{3}{4}$ the way up of total cum. freq (C = 100)

$\frac{1}{4}$ the way up

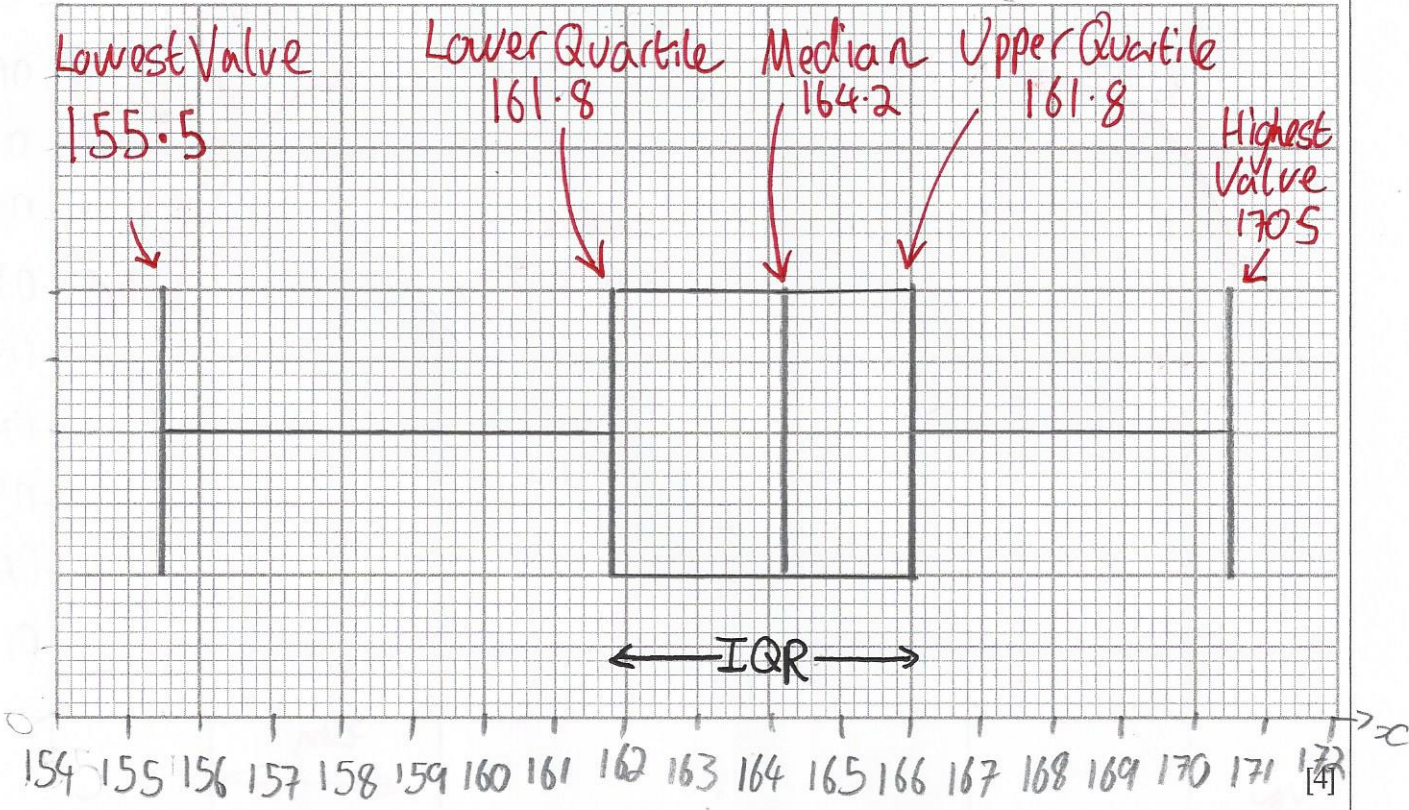*(c)* Use your cumulative frequency diagram to estimate the median and the interquartile range.

$\underline{IQR} = Upper\ Quartile - Lower\ Quartile$

$= \quad 166 \quad - \quad 161.8$

$= \quad 4.2$

Median ....164.2.... Interquartile range ....4.2....

[3]

(d) Use the graph paper below to draw a box-and-whisker diagram to show these results.

Lowest Value
155.5

Lower Quartile
161.8

Median
164.2

Upper Quartile
161.8

Highest Value
170 5

$\longleftarrow$ IQR $\longrightarrow$

154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172

[4]

Arm length in cm

Turn over.

Topic: Loci and Construction

7. A valuable statue is on display.

To protect the statue a glass cuboid is built around it.

A scale drawing of the plan view (bird's eye or aerial view) of the cuboid is shown below.
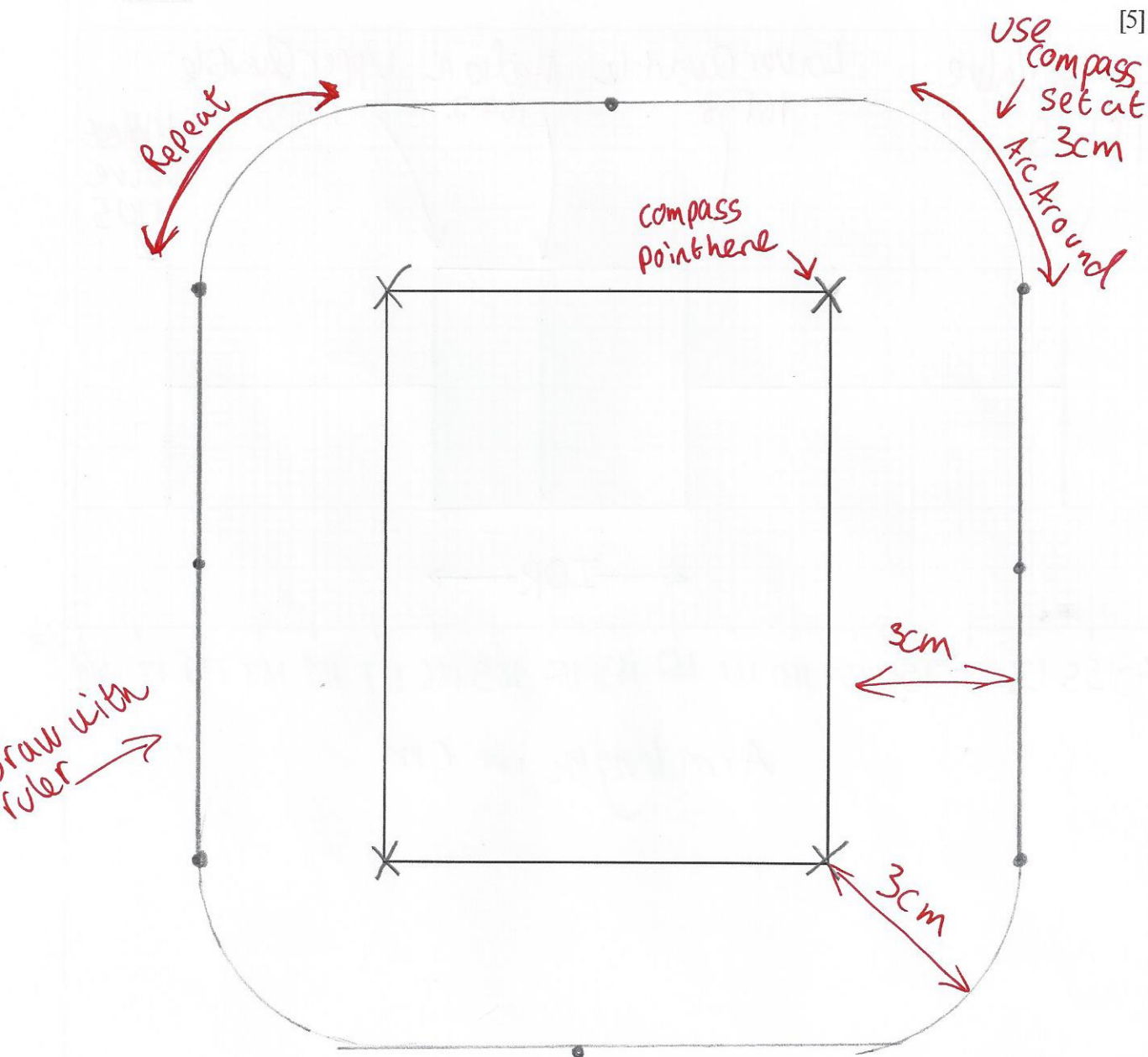
Scale 1 cm : 20 cm

A barrier is built around the cuboid so that no one can stand within 60 cm of the cuboid.

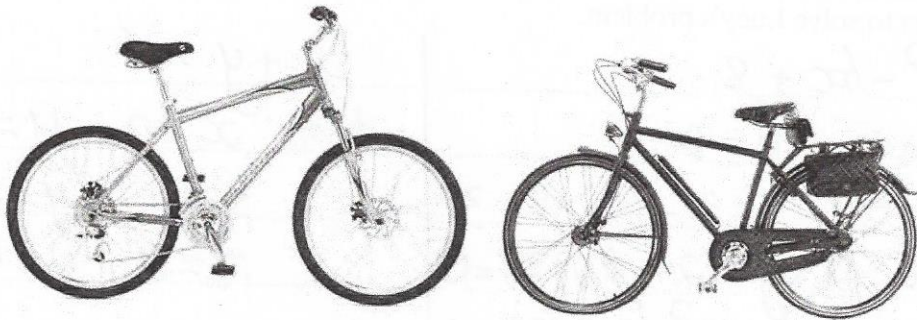Using the given scale, draw accurately the locus of the barrier on the scale drawing shown below.

[5]

Repeat

USE compass set at 3cm Arc Around

compass point here

3cm

3cm

3cm

draw with ruler

Scale 1cm : 20cm

=> 3cm : 60cm

(4361-02)

8. Tom and Zen have new bikes.
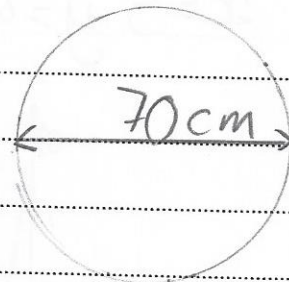
The diameters of the wheels on Tom's bike are 70 cm.
The diameters of the wheels on Zen's bike are 60 cm.

Calculate the difference in the number of revolutions of the wheels over a distance of 1 km.
Give your answer correct to 2 significant figures.

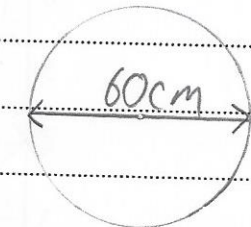*1 revolution is same as circumference*

Tom

Circumference of Circle $= \Pi D = 0.7 \Pi$ m

70 cm

Zen

Circumference of Circle $= \Pi D = 0.6 \Pi$ m

60 cm

1 km = 1000 m

Number of Tom revolution $= \dfrac{1000}{0.7 \Pi}$

How many times does circumference go into 1000m (1km)

Number of Zen revolution $= \dfrac{1000}{0.6 \Pi}$

Bigger number as smaller denominator

Difference in revolution $= \dfrac{1000}{0.6 \Pi} - \dfrac{1000}{0.7 \Pi}$

$= 75.78806\ldots$

$= 76$ to 2.s.f

[7]

(4361-02)

Turn over.
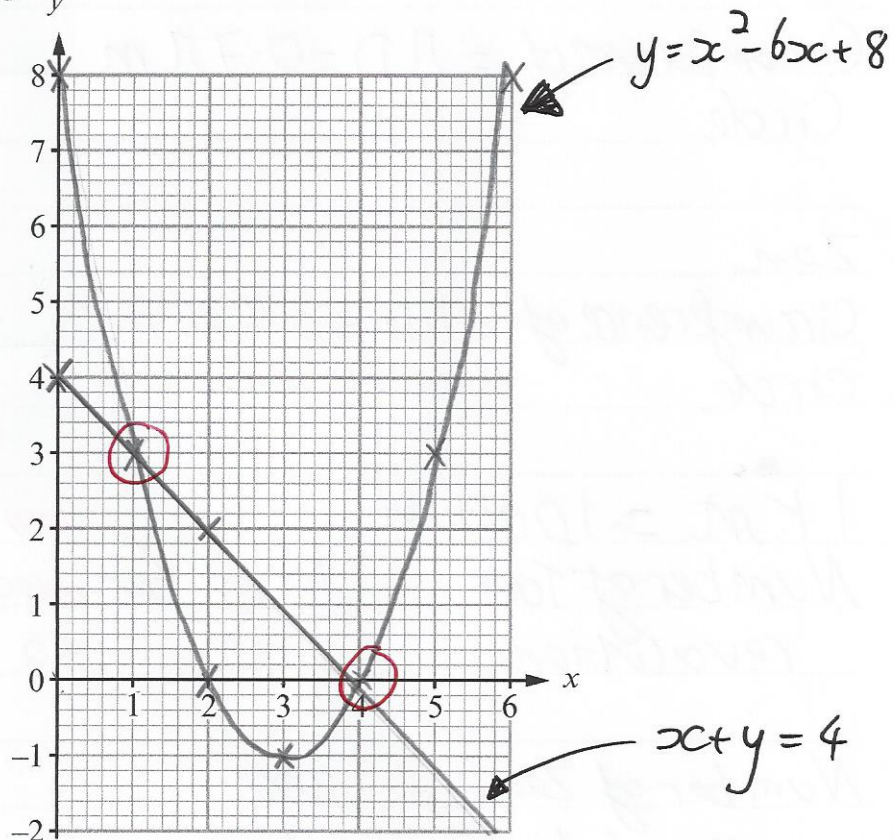
9. Lucy is a scientist. During an investigation she needs to find the points of intersection of two equations to solve a problem.
The equations are $y = x^2 - 6x + 8$ and $x + y = 4$.
Draw graphs to solve Lucy's problem.

$y = x^2 - 6x + 8$ $x + y = 4$

when $x = 0$ $y = 8$ when $x = 0$ $y = 4$

" $x = 1$ $y = 1^2 - 6(1) + 8 = 3$ $x = 1$ $y = 4 - 1 = 3$

" $x = 2$ $y = 2^2 - 6(2) + 8 = 0$ $x = 2$ $y = 4 - 2 = 2$

$x = 3$ $y = 3^2 - 6(3) + 8 = -1$

$x = 4$ $y = 4^2 - 6(4) + 8 = 0$

$x = 5$ $y = 5^2 - 6(5) + 8 = 3$

$x = 5$ $y = 6^2 - 6(6) + 8 = 8$

$y = x^2 - 6x + 8$

$x + y = 4$

[5]

Points of intersection $x = 1, y = 3$ and $x = 4, y = 0$
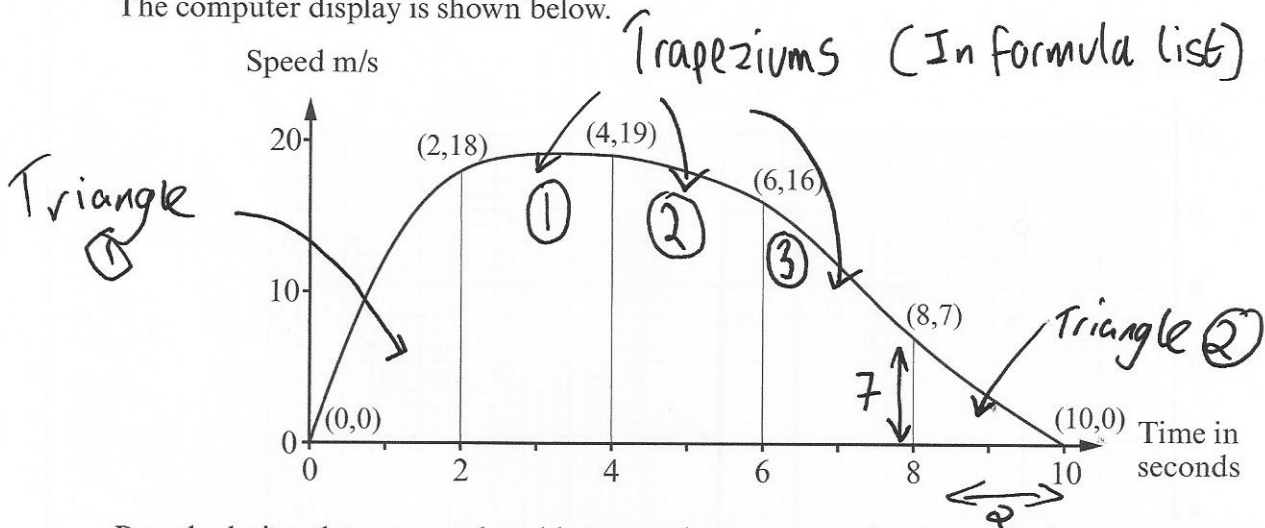
[2]

10. Dani is researching speed, distance and time.
    She carries out an experiment using a computer to generate a graph to show the speed of a particle over a 10 second time interval.
    The computer display is shown below.

Trapeziums (In formula list)

Triangle

Speed m/s

20 — (2,18)   (4,19)
                 ①   ②   (6,16)
                            ③
10 —
                              (8,7) *Triangle ②*
                         7↕
0 — (0,0)   (10,0) Time in seconds
   0   2   4   6   8   10

2

By calculating the area, enclosed between the curve and the time axis, Dani can estimate the distance the particle travelled.
Find an estimate for the distance travelled by the particle.
You must state the unit of your answer.

Area of triangle 1 = $\frac{1}{2}(b \times h) = \frac{1}{2}(2 \times 18) = 18m$

Area of triangle 2 = $\frac{1}{2}(b \times h) = \frac{1}{2}(2 \times 7) = 7m$

Area of Trapezium 1 = $\frac{1}{2}(a+b)h = \frac{1}{2}(18+19) \times 2 = 37m$

Area of Trapezium 2 = $\frac{1}{2}(a+b)h = \frac{1}{2}(19+16) \times 2 = 35m$

Area of Trapezium 3 = $\frac{1}{2}(a+b)h = \frac{1}{2}(16+7) \times 2 = 23m$

Total Distance = $18 + 7 + 37 + 35 + 23 = 120m$

metres [5]

$\frac{1}{x}$

11. The reciprocal of the speed of light squared is 2.22×10^{-18}.
    Find the speed of light in standard form correct to two significant figures.

$\frac{1}{V^2} = 2.22 \times 10^{-18}$

$\Rightarrow 1 = 2.22 \times 10^{-18} \times V^2$

$\Rightarrow V^2 = \dfrac{1}{2.22 \times 10^{-18}}$

standard form conversion [Press Eng on Calculator]

[3]

$\Rightarrow V = \sqrt{\dfrac{1}{2.22 \times 10^{-18}}} = 670 \times 10^6 \ ms^{-1}$
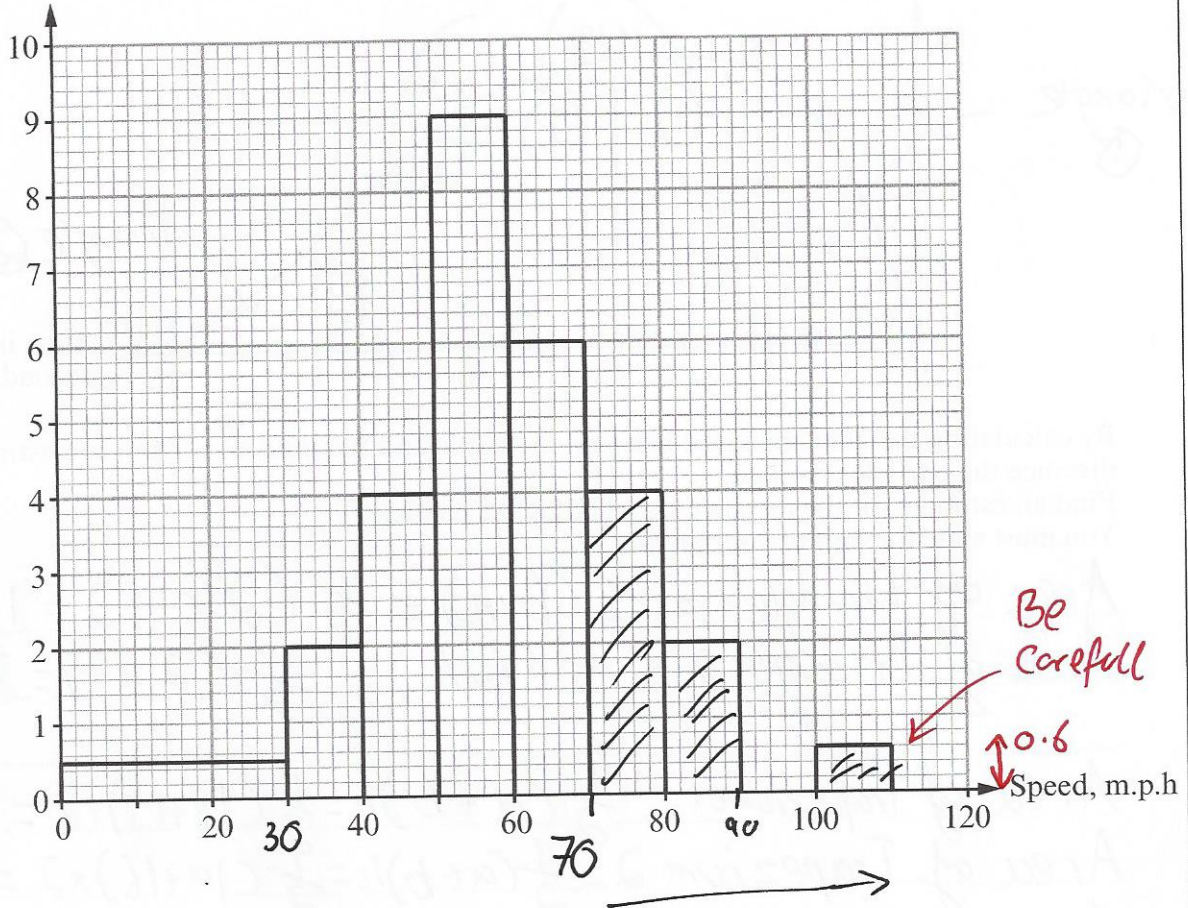
$= 6.7 \times 10^8 \ ms^{-1}$ 2.S.f

(4361-02) **Turn over.**

12. The histogram below shows the speeds of motorists as they enter a tunnel between 1 a.m. and 2 a.m.

Frequency density

Be
carefull

↑0·6

Speed, m.p.h

30

70

90

(a) The speed limit on entering the tunnel is 70 m.p.h.
How many motorists were exceeding the speed limit on entering the tunnel?

$Area > 70 = (10 \times 4) + (10 \times 2) + (10 \times 0.6)$

$Area = 66$

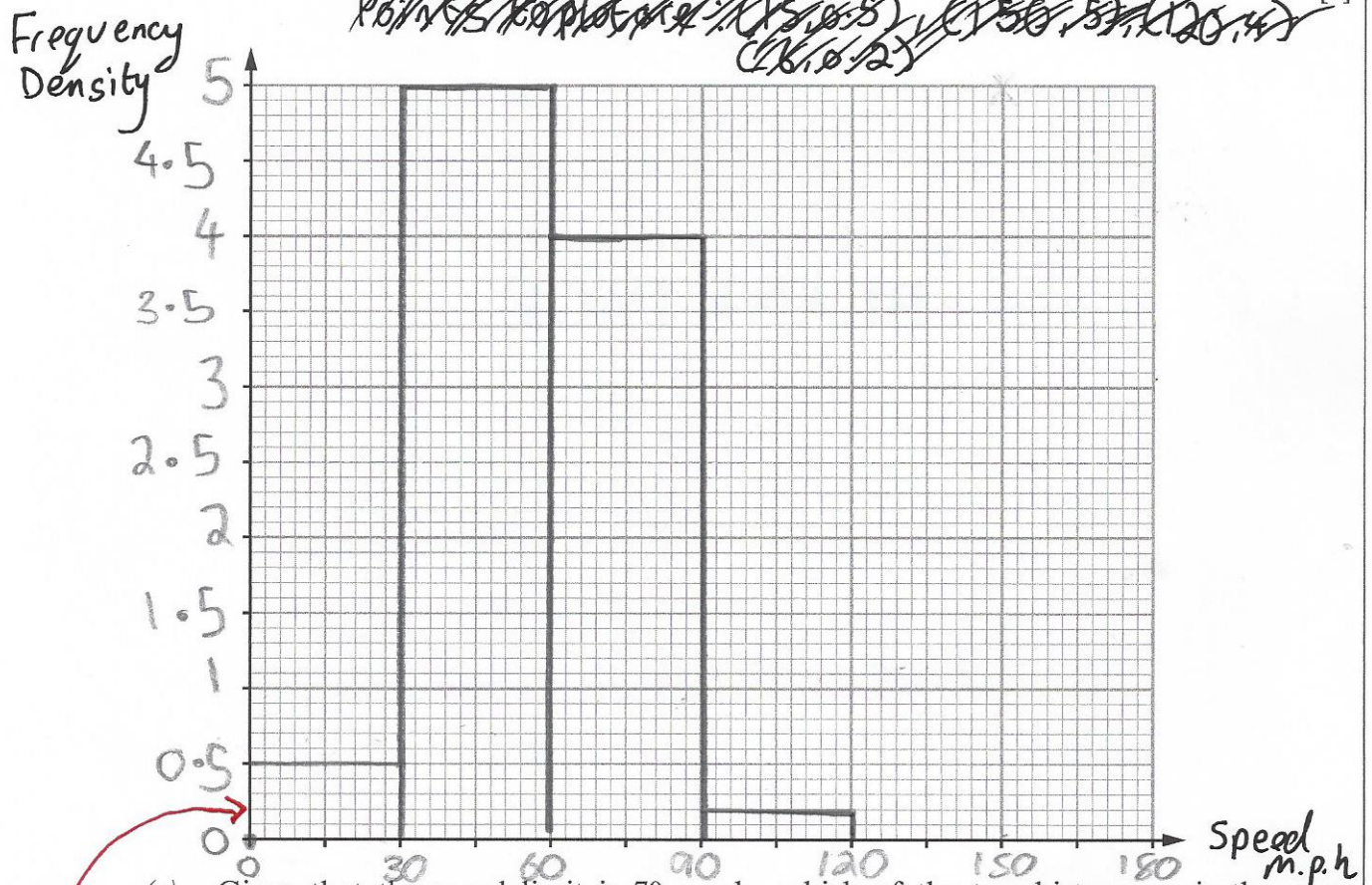$Area = number\ of\ people = 66$

[3]

Using Rule

Frequency Density = $\dfrac{\text{frequency}}{\text{Classwidth}}$ ← (motorists)

Examiner only

(b) In order to compare the speeds of motorists between 1 a.m. and 2 a.m. with other 1 hour periods, it is decided to group the data in equal intervals of width 30 m.p.h., starting at 0 m.p.h.
Construct a histogram to display these results meeting this new requirement.

Interval of 30 mph

① first Interval (0→30) = 30 × 0.5 = 15

② Second Interval (30→60) = (10×2)+(10×4)+(10×9) = 150

③ Third Interval (60→90) = (10×6)+(10×4)+(10×2) = 120

④ fourth Interval (90→120) = 10 × 0.6 = 6

Frequency Density ① = $\dfrac{15}{30}$ = 0.5   F.D ② = $\dfrac{150}{30}$ = 5

F.D ③ = $\dfrac{120}{30}$ = 4   F.D ④ = $\dfrac{6}{30}$ = 0.2

~~Points to plot are (15,0.5) (150,5) (120,4)~~ ~~(6,0.2)~~

[8]

Frequency Density (vertical axis: 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5)

Speed m.p.h (horizontal axis: 0, 30, 60, 90, 120, 150, 180)

(c) Given that the speed limit is 70 m.p.h., which of the two histograms is the more appropriate for the display of the data? You must give a reason for your answer.

1st histogram more appropriate as shows more intervals of speed, specifically 70, 80, 90, 100, 110, 120 onward, which are above speed limit

[1]

Circled numbers above shows why 0.5 is the height, if confused.